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Relationships Between Power of Exclusion and Probability of Paternity

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The fact that an alleged father is not excluded from paternity by a combination of genetic tests that would exclude, say, 999 out of 1000 randomly selected men is powerful intuitive evidence that the alleged father is, indeed, the biological father. Power of exclusion (the ability of a genetic test to exclude a falsely accused man of paternity) is thus a concept closely related to the concept of probability of paternity. Moreover, it is a concept that is more easily understood by those without mathematical training than is the Essen-Möller formulation of probability of paternity. This contribution will develop the concept of power of exclusion and the relationships between power of exclusion (A) and probability of paternity (W).

It will be argued that, while power of exclusion is an attractive concept, and a statistic from which one may derive an expected probability of paternity, only the probability of paternity (W) based on the Paternity Index (x/y) and the prior probability (p) is of direct relevance to the probability that an alleged father is, in fact, the biological father.

AVERAGE POWER OF EXCLUSION ($\bar{A}$)

For a particular genetic system, the average (or expected) power of exclusion ($\bar{A}$) is computed by summing over all mother-child phenotype combinations; for each mother-child combination, the proportion of men not fitting the description of the alleged father is computed. The average (or expected) power of exclusion is thus a function only of gene frequencies, and therefore of the ethnic background of the individuals to be tested. Methods for computing $\bar{A}$ and tabulations of $\bar{A}$ have been published for various genetic systems.[1, 2]

The average (or expected) power of exclusion for a combination of K systems is given as

$$\bar{A}=1-\prod_{j=1}^{k}(1-\bar{A}_j) \qquad \ldots (1)$$

Where $\bar{A}j$ is the average power of exclusion for the jth system.

The average power of exclusion of an individual genetic system is a figure of value to the laboratory for consideration of choice of methods; all other factors being equal, the laboratory should select genetic

systems with high expected powers of exclusion. The expected power of exclusion of a combination of systems provides the alleged father with his expectation (prior to testing of mother and child) of being excluded (provided he is not the father) by that combination of systems. The expected power of exclusion of systems available in a particular laboratory is a measure of the general ability of that laboratory to exclude falsely accused men. It is important to emphasize that once individuals in a disputed paternity matter have been phenotyped, the average power of exclusion of systems utilized no longer has relevance, and thus should not be part of the laboratory report.

THE (INDIVIDUAL) POWER OF EXCLUSION (A)

The individual power of exclusion depends on the phenotypes of mother and child, and the ethnic background of the alleged father. Unlike $\overline{A}$, A depends on the actual phenotypes of mother and child (Table 21-1).

The cumulative individual power of exclusion for a combination of genetic systems is given by a formula analogous to (l).

Unlike $\overline{A}$, A has direct relevance to a particular mother-child-alleged-father trio; A is the probability that a falsely accused alleged father would be excluded by the combination of genetic tests employed.

Power of exclusion undoubtedly has an intuitive relationship to probability of paternity; that it has a mathematical relationship as well can be seen from Fig 21-1, in which $\frac{1}{1-A}$ is plotted against the Paternity Index x/y. That there is good correlation over the entire range of values suggests that A contributes much of the information contained in x/y and that a probability of paternity may be derived from A. This is demonstrated in the next section.

Table 21-1—A Is a Strong Function of the Phenotypes of Mother and Child
ABO System; $\overline{A}$=0.15

Phenotypes		Power of Exclusion
Mother	Child	(A)
A	A	0
O	B	0.87
O	O	0.04

A DEFINES A PROBABILITY OF PATERNITY W_A[3, 4, 5]

Consider a mother-child pair of given phenotypes such that the power of exclusion equals A. Suppose there are K such pairs involved in "one-man" cases, with Kp fathers and K(1−p) nonfathers (ie, prior probability p), so that

$$AK(1-p) = \text{expected number of excluded men, and}$$

$$Kp+(1-A)K(1-p) = \text{expected number of nonexcluded men.}$$

Then, the expection of paternity (ie, probability of paternity) of a nonexcluded man

$$W_A = \frac{\text{expected number of fathers}}{\text{expected number of nonexcluded men}},$$

$$W_A = \frac{Kp}{Kp+K(1-A)(1-p)}$$

$$= \frac{p}{p+(1-A)(1-p)},$$

and

$$W_A = \frac{1}{2-A} \text{ (50\% prior probability).}$$

The odds in favor of paternity

$$= \frac{\text{fathers}}{\text{nonexcluded nonfathers}}$$

$$= \frac{Kp}{K(1-A)(1-p)}$$

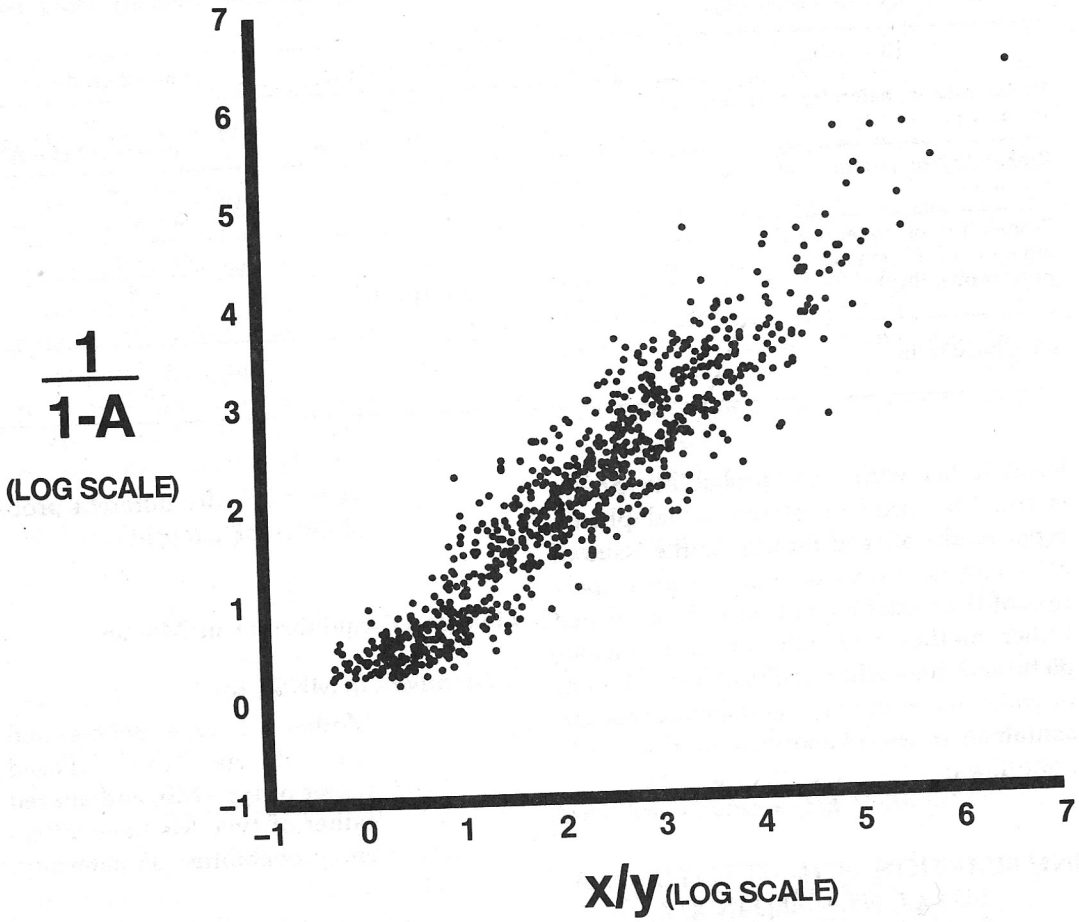

Fig 21-1—Relationship between Paternity Index (x/y) and $\frac{1}{1\text{-}A}$. Shown are 879 consecutive nonexclusion cases. Of these, 287 cases were tested in red cell enzymes and serum proteins only, 46 were tested in red cell antigens, enzymes and proteins, and 546 were tested in HLA, red cell antigens, enzymes and proteins.

and, at 50% prior probability, the odds in favor of paternity $=\frac{1}{1-A}$. This suggests that $\frac{1}{1-A}$ is a true likelihood ratio. Indeed, if we define W_A as the conditional probability $W_A = P(\text{paternity/p, nonexclusion})$. The corresponding likelihood ratio is:

$$\frac{P(\text{nonexclusion/paternity})}{P(\text{nonexclusion/nonpaternity})} = \frac{1}{1-A}.$$

Table 21-2 gives the familiar Bayesian formulas based on Paternity Index x/y and analogous formulas based on power of exclusion, A. It can be seen that the formulas have much the same structure. In particular, $\frac{1}{1-A}$ is analogous to the Paternity Index x/y; and both sets of formulations have the identical functional dependency on prior probability, p.

W_A has the following interpretation. Given testing in genetic systems with a power of exclusion of A, W_A gives the probability of paternity of a nonexcluded (but otherwise genetically undefined) al-

Table 21-2—Comparison of Expressions of Probability of Paternity Based on Paternity Index (x/y) and on Power of Exclusion (A)

Expression	Based on x/y	Based on A
Probability of paternity as a function of prior probability (p)	$W=\frac{p}{p+(1-p)\ y/x}$	$W_A=\frac{p}{p+(1-p)\ (1-A)}$
Probability of paternity at 50% prior probability	$W=\frac{1}{1+y/x}$	$W_A=\frac{1}{2-A}$
Probability of paternity for a combination of K genetic systems (50% prior probability)	$W=\frac{1}{1+\prod_{i=1}^{k} y_i/x_i}$	$W_A=\frac{1}{1+\prod_{i=1}^{k} (1-A_i)}$
Likelihood ratio	$PI=\frac{x}{y}$	$PI_A=\frac{1}{1-A}$

leged father with prior probability p; W_A is thus independent of the actual phenotypes of the alleged father. As the issue of exclusion is always determined by inspection of the actual phenotypes of the alleged father, mother, and child, W_A is an incomplete and somewhat artificial formulation, in which much of the available information contained in the phenotypes of the trio is not utilized.

INFORMATION NOT UTILIZED IN A, BUT CONTAINED IN X/Y

Gene Dosage in Alleged Father

Example: In Acid Phosphatase, for

Mother of type A and child of type A, alleged father of type A and alleged father of type AB have different probabilities of paternity.

Gene dosage is responsible for major differences in systems with a low, but known frequency of silent alleles.

Example: In Gc, given frequency of null allele = 0.001, for

Mother of type 1 and child of type 1, alleged father of type 1 and alleged father of type 2 have markedly different probabilities of paternity.

Linkage Disequilibrium in Mother

Example: In MNSs, for

Mother of type MNSs and child of type MNSs, alleged father of type MS, and alleged father of type Ms have different probabilities of paternity.

Linkage Disequilibrium in Alleged Father

Example: In Rh, for

Mother of type dCe and child of type dCe, alleged fathers of types DCce and alleged father of type DCEe have different probabilities of paternity.

OTHER PROPERTIES OF W_A

1. As $0 \leq A < 1$ $W_A \geq p$

Thus, W_A can never furnish evidence for nonpaternity. This is in sharp distinction to W. Very low values of W (approaching 0) may be obtained, despite the absence of exclusions.

2. At 50% prior probability,

$$W_A = \frac{1}{2-A} = \frac{1}{1+(1-A)}$$
$$= 1-(1-A)+(1-A)^2 \ldots$$

$W_A = A+(1-A)^2+\ldots$

$W_A > A$. Moreover, for $(1-A) \ll 1$

$W_A \approx A$.

For $A = 0.98$, $W_A = 0.9804$

3. W_A is the average probability of paternity ($\overline{W}$) for nonexcluded men with prior probability p tested in genetic systems with power of exclusion A.

For, consider K such cases. Then, we expect $Kp+K(1-p)(1-A)=n$ nonexcluded men. Calling their individual probabilities of paternity W_i, we put

$$\overline{W} = \frac{\sum_{i=1}^{n} W_i}{n}$$

but, ΣW_i = expected number of fathers $= Kp$ so that

$$\overline{W} = \frac{Kp}{Kp+K(1-p)(1-A)} = W_A$$

Note that while $\overline{W} = W_A$, the group of nonexcluded men is nonhomogeneous, consisting of Kp fathers and $K(1-p)(1-A)$ nonexcluded nonfathers. In general, $\overline{W}$ for fathers is greater than $\overline{W}$ for nonexcluded nonfathers.[6] The Paternity Index for each of these two groups is related to A in a simple way (see Chapter 11). The mean value of $\frac{y}{x}$ for fathers is equal to $1-A$, and the mean value of $\frac{x}{y}$ for nonexcluded nonfathers is equal to $\frac{1}{1-A}$.

4. From 2. and 3. above,

$\overline{W} > A$; For $(1-A) \ll 1$

$\overline{W} \approx A$ (50% prior probability).

Given phenotypes of mother and child such that the power of exclusion is A, one might argue that $W \geq A$ is "characteristic" of fathers, while $W < A$ is "characteristic" of nonfathers. This is nothing more than stating that the higher the value of W, the more likely paternity. (See Appendix.)

W is clearly to be preferred to W_A, as W makes use of all of the genetic information contained in the phenotypes of mother, child, and alleged father. On the most fundamental level probability of paternity is the expectation of paternity given the observed phenotypes and the prior probability. There can be no virtue in ignoring much of the genetic information contained in the observed phenotypes.

On a practical level, use of W_A does not allow one to distinguish among nonexcluded alleged fathers, and does not consider evidence for nonpaternity contained in low values of x/y. The relationship between W and W_A, based on clinical material, is shown in Fig 21-2. While there is good correlation, in individual cases W and W_A can be quite different. This can have significant legal consequences if testing is less extensive (Fig 21-3).

DOES A CONTAIN ADDITIONAL INFORMATION NOT CONTAINED IN W?

It has been argued [7] that consideration of both A and W may, in certain circumstances, provide a better estimate of probability of paternity than consideration of W alone. We have not found this to be the case in practice. (See Appendix.) Moreover, on a most fundamental level the functional form of the Bayesian formulation of probability of paternity—$W = f(p, x/y)$—indicates that all of the genetic information is contained in the ratio x/y (see Chapter 50). It would therefore be unexpected indeed if formulations of the genetic information contained in the phenotypes of the trio were of functional form involving A in addition to x/y.

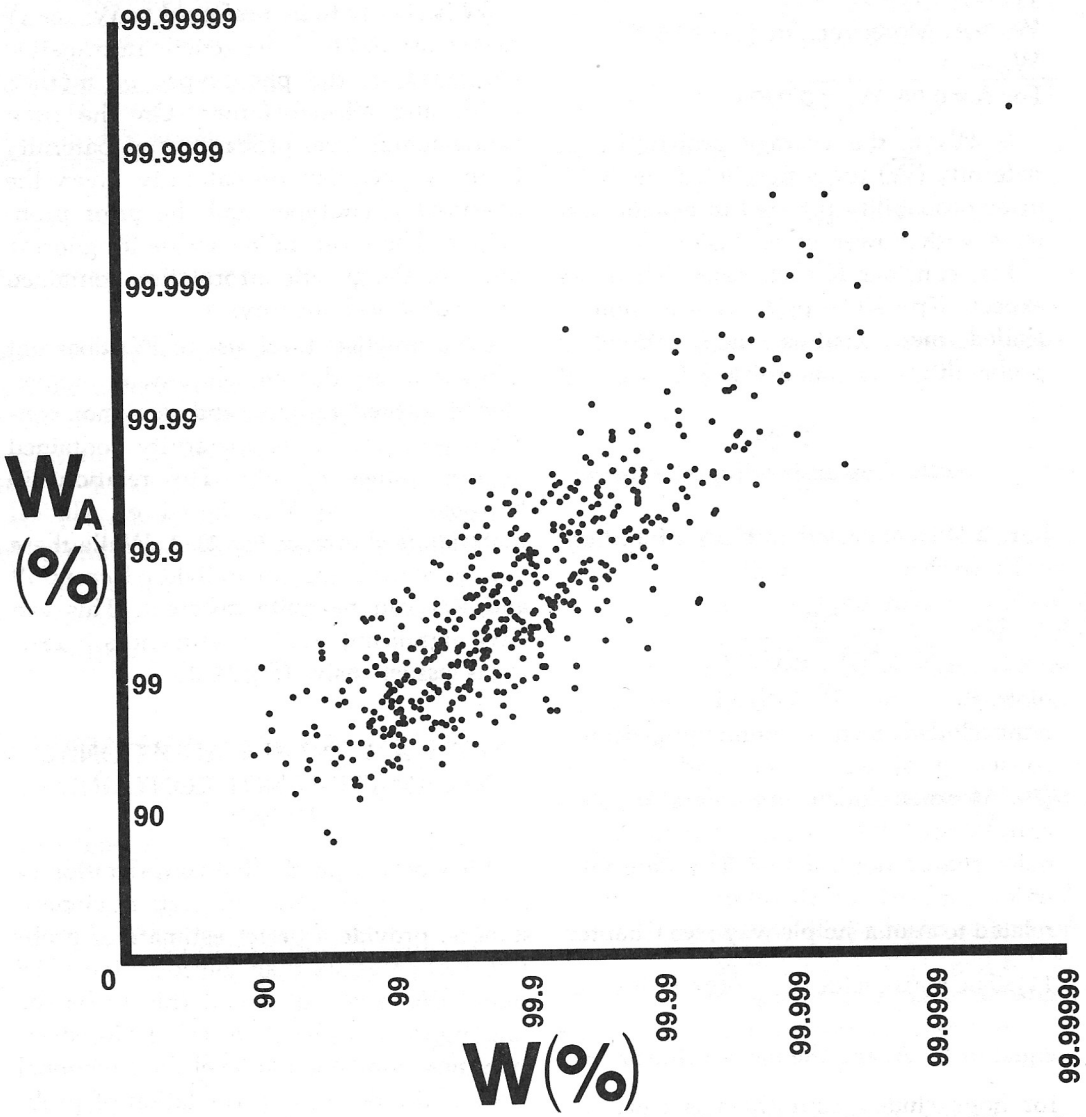

Fig 21-2—Relationship between probability of paternity computed on the basis of Paternity Index (x/y) and probability of paternity computed on the basis of power of exclusion (A). 50% prior probability was used. Plotted are 511 consecutive "one-man, one-child" cases in which no exclusions were found. Testing was done in red cell antigens, enzymes, proteins and HLA.

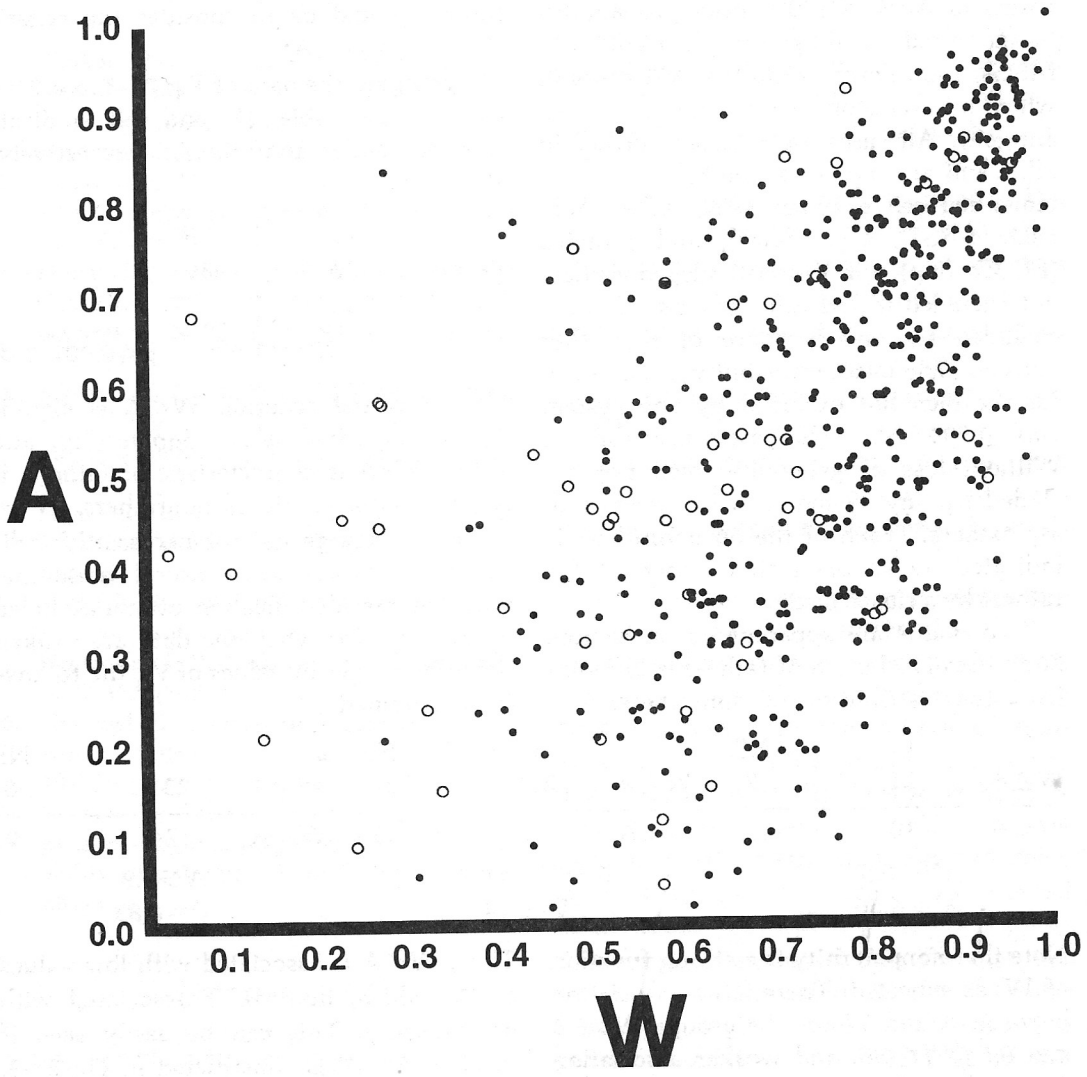

Fig 21-3—Relationship between probability of paternity at 50% prior probability (W) and power of exclusion (A) calculated on the basis of red cell antigen results. Shown are 601 consecutive cases in which no exclusions were found in red cell antigens and testing was also performed in proteins, enzymes and HLA. Alleged fathers excluded in non red cell antigen systems are shown by open circles. Alleged fathers not excluded by any system are shown by closed circles.

APPENDIX
RELATIONSHIP BETWEEN W AND A—CASE MATERIAL

Figure 21-3 gives a plot of calculated values of A vs W (50% prior probability) based on red cell antigen results (ABO, Rh, MNSs, Kell, Duffy, Kidd) for 601 cases in which no exclusions were found in red cell antigens. All cases were tested further in HLA and at least six of the following systems: red cell enzymes (AK, ADA, AcP, EsD, 6-PGD, Glo, PGM), and proteins (Bf, Gc, Hp). Of the 601 alleged fathers not excluded in red cell antigens, 56 were excluded by one or more of the other systems. The mean probability of paternity for the men not excluded by any system was 0.994 (at 50% prior probability). Without loss of generality, men not excluded by any system will be referred to as "fathers." Each of the 56 nonfathers is indicated by an open circle: each of the fathers by a closed circle.

Two points are apparent by inspection. First, the distribution of fathers is different from the distribution of nonfathers. Second, both the distribution of fathers and the distribution of nonfathers are nonsymmetric about the line A=W. One way of investigating the possible usefulness of A as an additional aid to identify nonfathers would be to consider the criteria $W \geq A$ and $W < A$.

Arranging the data of Fig 21-3 in a 2×2 contingency table (F and NF indicate fathers and nonfathers, respectively) shows:

	F	NF
$W \geq A$	473	39
$W < A$	72	17

$X^2 = 11.83$ $p < 0.001$, 1 df

Note that the criterion $W < A$ is significantly associated with nonpaternity, and while $W \geq A$ is characteristic of fathers, it is also characteristic of nonfathers.

These findings do not necessarily indicate that $W < A$ is a useful additional criterion for identification of nonexcluded nonfathers, for when the data are broken down by ranges of values of W, the following is obtained:

	F	NF
$W \geq A$	21	7
$W < A$	10	12

$W < 0.5$
$X^2 = 4.56$

	F	NF
$W \geq A$	221	24
$W < A$	38	5

$0.5 \leq W < 0.8$
$X^2 = 0.14$

	F	NF
$W \geq A$	231	8
$W < A$	24	0

$W > 0.8$
$X^2 = 0.83$

Note that nonpaternity is a strong function of W, as expected. There is no association between A and W for the groups $W > 0.8$ and $0.5 \leq W \leq 0.8$, and weaker association ($p < 0.05$) for $W < 0.5$. However, if the data for $W < 0.5$ are broken down further (into say, $W < 0.3$ and $0.3 \leq W \leq 0.5$), there is no longer any statistically significant association ($p > 0.05$). Moreover, for $W < 0.5$, the group $W \geq A$ can be distinguished from the group $W < A$ on the basis of $\overline{W}$:

for $W \geq A$, $\overline{W} = 0.417$ (n=28)
$W < A$, $\overline{W} = 0.333$ (n=22)

Thus, $W < A$ is associated with low values of W, which, in turn, is associated with nonpaternity. This can be easily seen if the line A=W is constructed in Fig 21-3. The great majority of low values of W occurs in the half-plane A>W.

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DISCUSSION

DR. NIJENHUIS: Dr. Morris' presentation contains a number of highlights and I will mention two of them. Both have to do with the direct relation that could be proved to exist between the chance of exclusion and the chance of paternity as calculated via the quotient of X/Y.

The first concerns the clear conclusion that the chance of exclusion does not contain or express any information that was not already incorporated in the Paternity Index. Reporting the chance of exclusion as an addition to the value of X/Y or W could suggest that this chance of exclusion provides additional information above that of the Paternity Index or the W value. In fact, such reporting is no more than a careless repetition of only a part of what has been said before, and with the omission and ignoration of an essential part of the relevant information.

A second aspect is the recognition of W_A to be equal to the mean value of W. This means, verbally expressed, that, given a certain phenotypical composition of the mother-child combination, the probability of paternity of any nonexcluded alleged father (either father or nonfather), as approached by the chance of nonexclusion of random nonfathers, is equal to the mean of the probabilities of paternity of these men, as approached by Essen-Möller's calculation.

Dr. Morris, by using two further equations—he was so kind to mention that he borrowed them from me—also showed for fathers and nonfathers apart that there is a direct relation with the chance of nonexclusion. I should like to add a little bit to this (see Fig 11-1). I values in fathers, as well as in nonexcluded nonfathers, are far from normally distributed. But, to some extent, they approach a logarithmic normal distribution.

When it is supposed that the distributions of the logarithms of the Paternity Index are really normal, the following picture is obtained (see Fig 11-1). In this graph, we will only consider the drawn curve, not the dotted curve. The dotted curve is not of interest for this purpose. The I value of c, below, at the crossing of the two curves, represents the value of 1/(the chance of nonexclusion). It is evident that in the left curve, the curve of the nonexcluded nonfathers, the greater part of its area is situated at the left of c, which

means that the greater part of nonexcluded nonfathers have I values, Paternity Index values, that are below 1/(the chance of nonexclusion). The greater part of the curve of true fathers concerns Paternity Index values above the value of 1/(the chance of nonexclusion). This means that, when probabilities of paternity would be based on the chance of nonexclusion, in the greater part of cases with nonfathers, the probability of paternity would be overestimated, and in the greater part of the cases with true fathers, the probability of paternity would be underestimated.

In summary, the major aspect of Dr. Morris' presentation concerns a final conclusion to be drawn, which is: Should it be advised, yes or no, to report, as an addition to the paternity index or W value, the chance of exclusion of nonfathers? In my opinion, it should not, because once the X/Y or W value has been reported, the chance of exclusion is absolutely without any relevance. To avoid confusion, it should be omitted. I should like to ask Dr. Morris whether he agrees with this opinion.

DR. MORRIS: We do not report the power of exclusion. We believe it would be confusing to report two numbers, and there is an overwhelming tendency for attorneys, when faced with both numbers, to use the inferior figure of power of exclusion. This can lead to serious errors.